The Homotopy Perturbation Method for Solving Nonlinear Burgers and New Coupled Modified Korteweg-de Vries Equations

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Z. Naturforsch. **63a,** 627 – 633 (2008); received April 23, 2008

We use the homotopy perturbation method to find the travelling wave solutions of nonlinear Burgers and new coupled modified Korteweg-de Vries equations. The results reveal that the homotopy perturbation method is very effective, convenient and quite accurate to systems of nonlinear equations. It is predicted that the homotopy perturbation method can find wide application in engineering and physics.

Key words: Homotopy Perturbation Method; Travelling Wave Solutions; Nonlinear Burgers and New Coupled MKdV Equations; Nonlinear Partial Differential Equations.

1. Introduction

Nonlinear partial differential equations are known to describe a wide variety of phenomena, not only in physics, where applications extend over magnetofluid dynamics, water surface gravity waves, electromagnetic radiation reactions, and ion acoustic waves in plasma, just to name a few, but also in biology and chemistry, and several other fields. It is one of the important tasks in the study of nonlinear partial differential equations to seek exact and explicit solutions. In the past several decades both mathematicians and physicists have made many attempts in this direction. Various methods for obtaining exact solutions to nonlinear partial differential equations have been proposed. Among these are the Bäcklund transformation method [1, 2], the Hirota's bilinear method [3], the inverse scattering transform method [4], the extended tanh method [5 – 7], the Adomian pade approximation [8 – 10], the variational method [11 – 14], the variational iteration method [15, 16], the various Lindstedt-Poincare methods [17 – 20], the Adomain decomposition method [8, 21, 22], the F-expansion method [23, 24], the Exp-function method [25 – 27] and others [28 – 35].

In the present paper, we will use the homotopy perturbation method to construct travelling wave solutions for the nonlinear Burgers equation [36]

$$u_t + uu_x = \beta u_{xx} \tag{1}$$

and the new nonlinear coupled modified Korteweg-de Vries (MKdV) system [35]

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3uv_x + 3vu_x - 3\lambda u_x, \\ v_t &= -v_{xxx} - 3vv_x - 3u_xv_x + 3u^2v_x + 3\lambda v_x, \end{aligned} \tag{2}$$

where β and λ are arbitrary constants. The homotopy perturbation method was first proposed by He [37 – 46]. It does not depend on a small parameter. Using the homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0, 1]$ which is considered as a small parameter. Recently, the Burgers equation (1) has been solved in [47, 48] using the Adomain method. On comparing the Adomain method used in [47, 48] with the homotopy perturbation method used in the present paper, we conclude that the homotopy perturbation method is more effective and can overcome the difficulties arising in complex calculation of Adomain polynomials.

2. Applications

In this section, we discuss the travelling wave solutions of the Burgers equation (1) and the new coupled

MKdV equations (2) by using the homotopy perturbation method. Applications of this method to similar equations can be found in [49–51].

2.1. The Homotopy Perturbation Method for the Burgers Equation

In this subsection, we find the solution $u(x,t)$ satisfying the nonlinear Burgers equation (1) with the initial condition [36]

$$u(x,0) = c\left[1-\tanh\left(\frac{cx}{2\beta}\right)\right], \tag{3}$$

where c is a constant.

Following the homotopy perturbation method proposed by He [37–46], we construct a homotopy V for the Burgers equation (1) which satisfies

$$(1-p)(V^{\cdot}-\dot{u_0})+p(V^{\cdot}+VV^{\prime}-\beta V^{\prime\prime})=0 \tag{4}$$

with the initial approximation

$$V_0(x,t)=u_0(x,t)=u(x,0), \tag{5}$$

while

$$V=V_0+pV_1+p^2V_2+p^3V_3+\ldots, \tag{6}$$

where V_j $(j=1,2,3,\ldots)$ are functions to be determined and $p\in[0,1]$. In (4) the "primes" denote differentiation with respect to x, and the "dot" denote differentiation with respect to t. Substituting (5) and (6) into (4) and arranging the coefficients of the powers of p, we have

$$\begin{aligned}&[\dot{V_1}+V_0V_0^{\prime}-\beta V_0^{\prime\prime}]p+[\dot{V_2}+V_0V_1^{\prime}+V_1V_0^{\prime}-\beta V_1^{\prime\prime}]p^2\\&+[\dot{V_3}+V_0V_2^{\prime}+V_1V_1^{\prime}+V_2V_0^{\prime}-\beta V_2^{\prime\prime}]p^3+\ldots=0.\end{aligned} \tag{7}$$

In order to obtain the unknowns $V_j(x,t)$ $(j=1,2,3,\ldots)$ we construct and solve the following system which includes three equations with three unknown variables considering the initial condition (3):

$$\begin{aligned}&\dot{V_1}+V_0V_0^{\prime}-\beta V_0^{\prime\prime}=0,\\&\dot{V_2}+V_0V_1^{\prime}+V_1V_0^{\prime}-\beta V_1^{\prime\prime}=0,\\&\dot{V_3}+V_0V_2^{\prime}+V_1V_1^{\prime}+V_2V_0^{\prime}-\beta V_2^{\prime\prime}=0.\end{aligned} \tag{8}$$

From (6), if the first three approximations are sufficient, we obtain

$$u(x,t)=\lim_{p\to 1}V(x,t)=\sum_{k=0}^{3}V_k(x,t), \tag{9}$$

where

$$V_0(x,t)=c\left[1-\tanh\left(\frac{cx}{2\beta}\right)\right], \tag{10}$$

$$V_1(x,t)=\frac{c^3t}{2\beta}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right), \tag{11}$$

$$V_2(x,t)=\frac{c^5t^2}{4\beta^2}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right)\tanh\left(\frac{cx}{2\beta}\right), \tag{12}$$

$$\begin{aligned}V_3(x,t)=&-\frac{c^7t^3}{8\beta^3}\operatorname{sech}^4\left(\frac{cx}{2\beta}\right)\\&+\frac{c^7t^3}{12\beta^3}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right).\end{aligned} \tag{13}$$

Substituting (10)–(13) into (9), we obtain the approximate solution

$$\begin{aligned}u(x,t)=&\,c\left[1-\tanh\left(\frac{cx}{2\beta}\right)\right]+\frac{c^3t}{2\beta}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right)\\&+\frac{c^5t^2}{4\beta^2}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right)\tanh\left(\frac{cx}{2\beta}\right)\\&-\frac{c^7t^3}{8\beta^3}\operatorname{sech}^4\left(\frac{cx}{2\beta}\right)\\&+\frac{c^7t^3}{12\beta^3}\operatorname{sech}^2\left(\frac{cx}{2\beta}\right)+\ldots.\end{aligned} \tag{14}$$

Using the Taylor series, we obtain the closed form solution as follows:

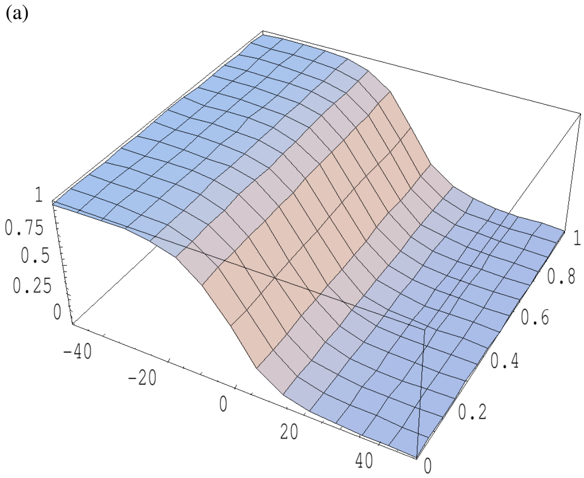
$$u(x,t)=c\left[1-\tanh\left(\frac{c}{2\beta}(x-ct)\right)\right]. \tag{15}$$

With the initial condition (3), the solitary wave solution of the Burgers equation (1) of the bell-type for $u(x,t)$ is in full agreement with the ones constructed by Wazwaz [36]. To demonstrate the convergence of the homotopy perturbation method, the results of the numerical example are presented and only few terms are required to obtain accurate solutions. The accuracy of the homotopy perturbation method for the nonlinear Burgers equation is controllable, and absolute errors are very small with the present choice of x,t. These results are listed in the Table 1. It can be seen that the implemented method achieves a minimum accuracy of four and a maximum accuracy of eleven significant figures for (1) for the first three approximations. Both the exact solution and approximate solution obtained for the first approximation are plotted in Figure 1. There

Table 1. The homotopy perturbation result of $u(x,t)$ for the first three approximations in comparison with the analytical solution, if $c = 0.5$ and $\beta = 3$, for the solitary wave solutions with the initial condition (3) of the Burgers equation (1).

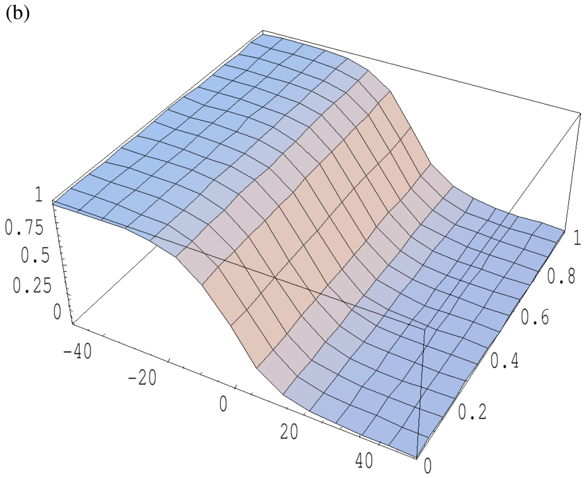
(x,t)	u_{homotopy}	u_{exact}	$\lvert u_{\text{exact}} - u_{\text{homotopy}}\rvert$
$(0.1,0.1)$	0.497917	0.497917	$7.53397 \cdot 10^{-13}$
$(0.1,0.2)$	0.5	0.5	$1.071521 \cdot 10^{-11}$
$(0.1,0.3)$	0.502083	0.502083	$4.7465698 \cdot 10^{-11}$
$(0.2,0.1)$	0.49375	0.49375	$1.5896 \cdot 10^{-12}$
$(0.2,0.2)$	0.495833	0.495833	$2.40970 \cdot 10^{-11}$
$(0.2,0.3)$	0.497917	0.497917	$1.15219 \cdot 10^{-10}$
$(0.3,0.1)$	0.489585	0.489585	$2.4239 \cdot 10^{-12}$
$(0.3,0.2)$	0.491667	0.491667	$3.74502 \cdot 10^{-11}$
$(0.3,0.3)$	0.49375	0.49375	$1.8283 \cdot 10^{-10}$

(a)



u_{homotopy}

(b)



u_{exact}

Fig. 1. (a) The homotopy perturbation result of $u(x,t)$ for the first three approximations. (b) The analytical solution, if $c = 0.5$ and $\beta = 3$, for the solitary wave solutions with the initial condition (3) of the Burgers equation (1).

are no visible differences in the diagrams. It is also evident that, when more terms for the homotopy perturbation method are computed, the numerical result gets much more closer to the corresponding exact solution with the initial condition (3).

2.2. The Homotopy Perturbation Method for New Coupled MKdV Equations

In this subsection, we find the solutions $u(x,t)$ and $v(x,t)$ satisfying the new coupled MKdV equations (2) with the initial conditions [35]

$$u(x,0) = \frac{b_1}{2k} + k\tanh(kx)$$

and

$$v(x,0) = \frac{\lambda}{2}\left(1 + \frac{k}{b_1}\right) + b_1 \tanh(kx), \tag{16}$$

where k, b_1 and λ are arbitrary constants.

With reference to the homotopy perturbation method proposed by He [37–46], we construct two homotopies v_1 and v_2 for the new coupled MKdV equations (2) which satisfy

$$(1-p)(\dot{v}_1 - \dot{u}_0) + p\Big(\dot{v}_1 - \frac{1}{2}v_1''' + 3v_1^2 u_1' - \frac{3}{2}v_2'' - 3v_1 v_2' - 3v_1' v_2 + 3\lambda v_1'\Big) = 0$$

and

$$(1-p)(\dot{v}_2 - \dot{v}_0) + p\big(\dot{v}_2 + v_2''' + 3v_2 v_2' + 3v_1' v_2' - 3v_1^2 v_2' - 3\lambda v_2'\big) = 0 \tag{17}$$

with the initial approximations

$$v_{1,0}(x,t) = u_0(x,t) = u(x,0)$$

and

$$v_{2,0}(x,t) = v_0(x,t) = v(x,0), \tag{18}$$

while

$$v_1 = v_{1,0} + p v_{1,1} + p^2 v_{1,2} + p^3 v_{1,3} + \ldots$$

and

$$v_2 = v_{2,0} + p v_{2,1} + p^2 v_{2,2} + p^3 v_{2,3} + \ldots, \tag{19}$$

where $v_{i,j}$ ($i = 1,2,\ j = 1,2,3,\ldots$) are functions to be determined and $p \in [0,1]$. Substituting (18) and (19) into (17) and arranging the coefficients of the powers p, we have

$$\begin{aligned}&\Big[\dot{v}_{1,1}-\dot{u}_0-\frac{1}{2}v'''_{1,0}+3v_{1,0}^2v'_{1,0}-\frac{3}{2}v''_{2,0}\\&-3v_{1,0}v'_{2,0}-3v_{2,0}v'_{1,0}+3\lambda v'_{1,0}\Big]p\\&+\Big[\dot{v}_{1,2}-\frac{1}{2}v'''_{1,1}+3v_{1,0}^2v'_{1,1}+6v_{1,0}v_{1,1}v'_{1,0}-\frac{3}{2}v''_{2,1}\\&-3v_{1,0}v'_{2,1}-3v_{1,1}v'_{2,0}+3\lambda v'_{1,1}\Big]p^2+\ldots=0,\end{aligned}\tag{20}$$

$$\begin{aligned}&\big[\dot{v}_{2,1}-\dot{v}_0+v'''_{2,0}+3v_{2,0}v'_{2,0}+3v'_{1,0}v'_{2,0}\\&-3v_{1,0}^2v'_{2,0}+3\lambda v'_{2,0}\big]p\\&+\big[\dot{v}_{2,2}+v'''_{2,1}+3v_{2,1}v'_{2,0}+3v_{2,0}v'_{2,1}\\&+3v'_{1,0}v'_{2,1}+3v'_{1,1}v'_{2,0}-6v_{1,1}v_{1,0}v'_{2,0}\\&-3v_{1,0}^2v'_{2,1}-3\lambda v'_{2,1}\big]p^2\ldots=0.\end{aligned}\tag{21}$$

In order to obtain the unknowns $v_{i,j}(x,t)$ $(i=j=1,2)$ we construct and solve the following system which includes four equations with four unknown variables considering the initial conditions (16):

$$\begin{aligned}&\dot{v}_{1,1}-\dot{u}_0-\frac{1}{2}v'''_{1,0}+3v_{1,0}^2v'_{1,0}-\frac{3}{2}v''_{2,0}-3v_{1,0}v'_{2,0}\\&-3v_{2,0}v'_{1,0}+3\lambda v'_{1,0}=0,\\&\dot{v}_{1,2}-\frac{1}{2}v'''_{1,1}+3v_{1,0}^2v'_{1,1}+6v_{1,0}v_{1,1}v'_{1,0}-\frac{3}{2}v''_{2,1}\\&-3v_{1,0}v'_{2,1}-3v_{1,1}v'_{2,0}+3\lambda v'_{1,1}=0,\\&\dot{v}_{2,1}-\dot{v}_0+v'''_{2,0}+3v_{2,0}v'_{2,0}+3v'_{1,0}v'_{2,0}\\&-3v_{1,0}^2v'_{2,0}+3\lambda v'_{2,0}=0,\\&\dot{v}_{2,2}+v'''_{2,1}+3v_{2,1}v'_{2,0}+3v_{2,0}v'_{2,1}+3v'_{1,0}v'_{2,1}\\&+3v'_{1,1}v'_{2,0}-6v_{1,1}v_{1,0}v'_{2,0}-3v_{1,0}^2v'_{2,1}-3\lambda v'_{2,1}=0.\end{aligned}\tag{22}$$

From (19), if the first two approximations are sufficient, we obtain

$$u(x,t)=\lim_{p\to 1}v_1(x,t)=\sum_{k=0}^{1}v_{1,k}(x,t)\tag{23}$$

and

$$v(x,t)=\lim_{p\to 1}v_2(x,t)=\sum_{k=0}^{1}v_{2,k}(x,t),\tag{24}$$

where

$$v_{1,0}(x,t)=\frac{b_1}{2k}+k\tanh(kx),\tag{25}$$

$$v_{1,1}(x,t)=\frac{tk^2}{4}\left(-4k^2-6\lambda+\frac{6\lambda k}{b_1}+\frac{3b_1^2}{k^2}\right)\operatorname{sech}^2(kx),\tag{26}$$

$$v_{2,0}(x,t)=\frac{\lambda}{2}\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx),\tag{27}$$

$$v_{2,1}(x,t)=\frac{tb_1k}{4}\left(-4k^2+6\lambda-\frac{6\lambda k}{b_1}+\frac{3b_1^2}{k^2}\right)\operatorname{sech}^2(kx).\tag{28}$$

In view of (23) and (24), the solutions in a series form are given by

$$u(x,t)=v_{1,0}(x,t)+v_{1,1}(x,t)+\ldots$$

and

$$v(x,t)=v_{2,0}(x,t)+v_{2,1}(x,t)+\ldots.\tag{29}$$

In this manner, the approximate solutions of the new coupled MKdV system (2) take the following form:

$$\begin{aligned}u(x,t)=&\frac{b_1}{2k}+k\tanh(kx)\\&+\frac{tk^2}{4}\left(-4k^2-6\lambda+\frac{6\lambda k}{b_1}+\frac{3b_1^2}{k^2}\right)\operatorname{sech}^2(kx)\\&+\ldots\end{aligned}\tag{30}$$

and

$$\begin{aligned}v(x,t)=&\frac{\lambda}{2}\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx)\\&+\frac{tb_1k}{4}\left(-4k^2+6\lambda-\frac{6\lambda k}{b_1}+\frac{3b_1^2}{k^2}\right)\operatorname{sech}^2(kx)\\&+\ldots.\end{aligned}\tag{31}$$

With reference to [35], the exact solutions of the system (2) take the following form:

$$u(x,t)=\frac{b_1}{2k}+k\tanh(k\xi)\tag{32}$$

and

$$v(x,t)=\frac{\lambda}{2}\left(1+\frac{k}{b_1}\right)+b_1\tanh(k\xi),\tag{33}$$

where

$$\xi=x+\frac{t}{4}\left(-4k^2-6\lambda+\frac{6k\lambda}{b_1}+\frac{3b_1^2}{k^2}\right).$$

The comparisons between the exact solutions (32) and (33) and the approximate solutions (30) and (31) are shown in Table 2 and Figure 2. It seems that the errors are very small, if $b_1=0.1$, $k=0.1$, $t=0.5$ and $\lambda=0$.

x	u_{exact}	u_{homotopy}	$\lvert u_{\text{exact}} - u_{\text{homotopy}}\rvert$	v_{exact}	v_{homotopy}	$\lvert v_{\text{exact}} - v_{\text{homotopy}}\rvert$
-50	0.40001	0.40001	$2.5481 \cdot 10^{-8}$	-0.0999902	-0.0999902	$2.5481 \cdot 10^{-8}$
-40	0.400072	0.400072	$1.88057 \cdot 10^{-6}$	-0.0999278	-0.099928	$1.88057 \cdot 10^{-7}$
-30	0.400532	0.400531	$1.3774 \cdot 10^{-6}$	-0.099469	-0.0994676	$1.37741 \cdot 10^{-6}$
-20	0.403868	0.403859	$9.54085 \cdot 10^{-5}$	-0.0961318	-0.0961413	$9.54085 \cdot 10^{-6}$
-10	0.425439	0.425394	$4.43067 \cdot 10^{-5}$	-0.0745612	-0.0746055	$4.43067 \cdot 10^{-5}$
0	0.503698	0.5037	$1.68751 \cdot 10^{-6}$	0.00369831	0.0037	$1.68751 \cdot 10^{-6}$
10	0.57767	0.577713	$4.32578 \cdot 10^{-5}$	0.0776701	0.0777133	$4.32578 \cdot 10^{-5}$
20	0.596655	0.596664	$9.11419 \cdot 10^{-6}$	0.0966551	0.0966642	$9.11419 \cdot 10^{-6}$
30	0.599541	0.599542	$1.31174 \cdot 10^{-6}$	0.0995407	0.099542	$1.31174 \cdot 10^{-6}$
40	0.599938	0.599938	$1.79016 \cdot 10^{-7}$	0.0999377	0.0999379	$1.79016 \cdot 10^{-7}$
50	0.599992	0.599992	$2.42546 \cdot 10^{-7}$	0.0999916	0.0999916	$2.42546 \cdot 10^{-8}$

Table 2. The homotopy perturbation results of $u(x,t)$ and $v(x,t)$ for the first two approximations in comparison with the analytical solutions, if $\lambda = 0$, $b_1 = 0.1$, $t = 0.5$, and $k = 0.1$, for the solitary wave solutions with the initial conditions (16) of (2).

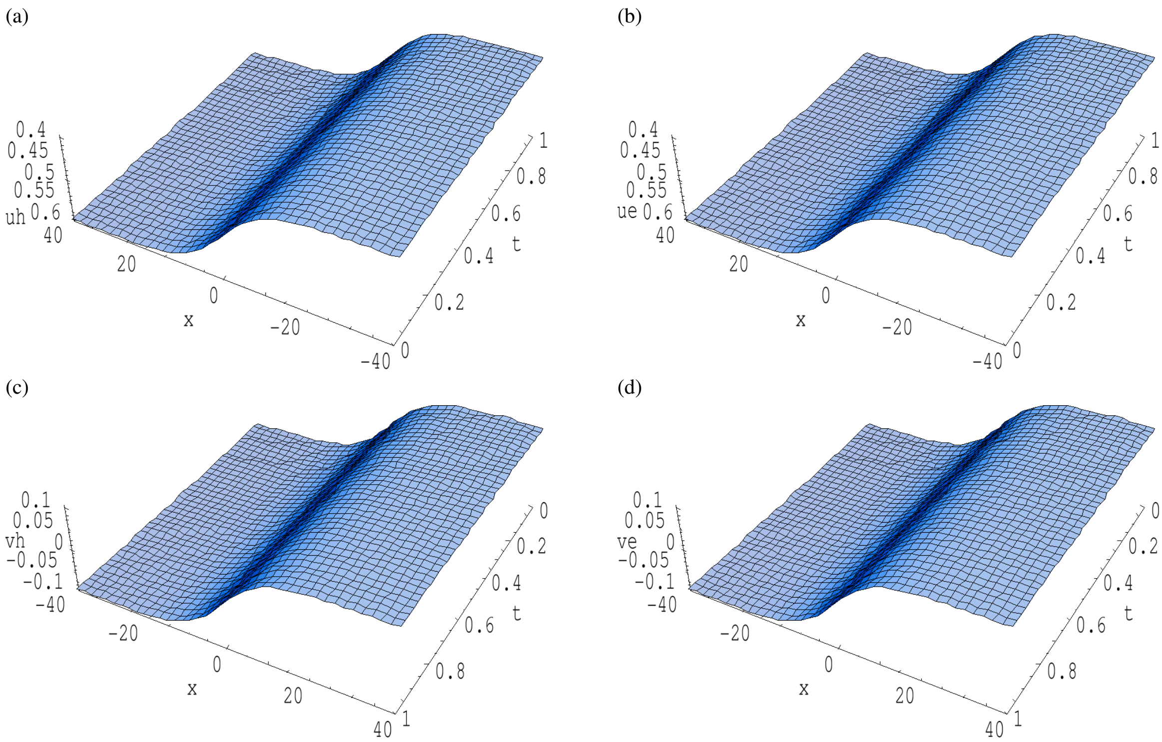


Fig. 2. The homotopy perturbation results of $u(x,t)$ and $v(x,t)$ for the first two approximations shown in (b) and (d) in comparison with the analytical solutions (a) and (c), if $\lambda = 0$, $b_1 = 0.1$, and $k = 0.1$, for the solitary wave solutions with the initial conditions (16) of (2).

Furthermore, to examine the accuracy and reliability of the homotopy perturbation method for the new coupled MKdV equations (2), we can also consider the different initial conditions [35]

$$u(x,0) = k\tanh(kx)$$

and

$$v(x,0) = \frac{1}{2}(4k^2 + \lambda) - 2k^2\tanh^2(kx). \tag{34}$$

After some reduction, we get

$$u_1(x,t) = \frac{t^2k^2}{2}(-3\lambda - 2k^2)\operatorname{sech}^2(kx) \tag{35}$$

and

$$v_1(x,t) = 2k^3t(2k^2 - 3\lambda)\operatorname{sech}^2(kx)\tanh(kx). \tag{36}$$

Consequently, the approximate solutions are

$$u(x,t) = k\tanh(kx) + \frac{t^2k^2}{2}(-3\lambda - 2k^2)\operatorname{sech}^2(kx) + \dots \tag{37}$$

x	u_{exact}	$u_{homotopy}$	$\lvert u_{exact} - u_{homotopy}\rvert$	v_{exact}	$v_{homotopy}$	$\lvert v_{exact} - v_{homotopy}\rvert$
−50	−0.0999922	−0.0999923	$9.84799 \cdot 10^{-8}$	0.500003	0.500004	$1.05002 \cdot 10^{-6}$
−40	−0.0999423	−0.0999431	$7.2687 \cdot 10^{-7}$	0.500023	0.500031	$7.74993 \cdot 10^{-7}$
−30	−0.0995746	−0.09958	$5.32722 \cdot 10^{-6}$	0.50017	0.500227	$5.67927 \cdot 10^{-5}$
−20	−0.0968991	−0.0969362	$3.94855 \cdot 10^{-4}$	0.501221	0.501616	$3.70688 \cdot 10^{-5}$
−10	−0.0791524	−0.0793302	$1.77785 \cdot 10^{-4}$	0.50747	0.509353	$1.88286 \cdot 10^{-3}$
0	−0.0075356	−0.00755	$1.4313 \cdot 10^{-5}$	0.519886	0.52	$1.13573 \cdot 10^{-4}$
10	0.0728019	0.0729886	1.866810^{-4}	0.5094	0.507446	$1.95342 \cdot 10^{-3}$
20	0.0958286	0.0958693	$4.06956 \cdot 10^{-5}$	0.501634	0.50121	$4.2369 \cdot 10^{-4}$
30	0.0994251	0.099431	$5.88558 \cdot 10^{-6}$	0.500229	0.500168	$6.12332 \cdot 10^{-5}$
40	0.099922	0.0999228	$8.03749 \cdot 10^{-7}$	0.500031	0.500023	$8.36135 \cdot 10^{-6}$
50	0.0999894	0.0999895	$1.08909 \cdot 10^{-7}$	0.500004	0.500003	$1.13296 \cdot 10^{-6}$

Table 3. The homotopy perturbation results of $u(x,t)$ and $v(x,t)$ for the first two approximations in comparison with the analytical solutions, if $\lambda = 0.5$, $b_1 = 0.1$, $t = 0.5$, and $k = 0.1$, for the solitary wave solutions with the initial conditions (34) of (2).

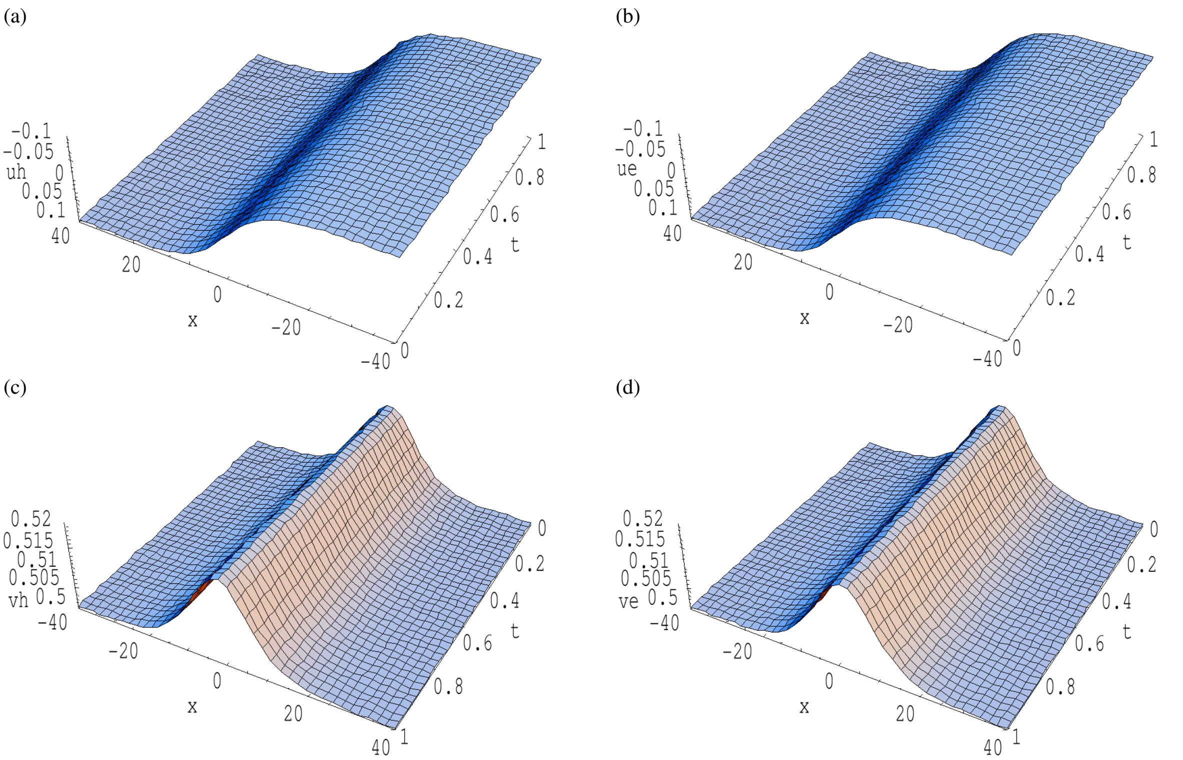


Fig. 3. The homotopy perturbation results of $u(x,t)$ and $v(x,t)$ for the first two approximations shown in (a) and (c) in comparison with the analytical solutions (b) and (d), if $\lambda = 0.5$, $b_1 = 0.1$, and $k = 0.1$, for the solitary wave solutions with the initial conditions (34) of (2).

and

$$v(x,t) = \frac{1}{2}(4k^2+\lambda) - 2k^2\tanh^2(kx) + 2k^3t(2k^2-3\lambda)\operatorname{sech}^2(kx)\tanh(kx) + \dots. \tag{38}$$

With reference to [35], the exact solutions of the system (2) take the following form:

$$u(x,t) = k\tanh(k\xi) \tag{39}$$

and

$$v(x,t) = \frac{1}{2}(4k^2+\lambda) - 2k^2\tanh^2(k\xi), \tag{40}$$

where

$$\xi = x + \frac{t}{2}(-2k^2-3\lambda).$$

The comparisons between the exact solutions (39) and (40) and the approximate solutions (37) and (38) are shown in Table 3 and Figure 3. It seems that the errors are very small, if $b_1 = 0.1$, $k = 0.1$, $t = 0.5$ and $\lambda = 0.5$.

3. Conclusions

In the present paper, the homotopy perturbation method was used to find the soliton wave solutions of the Burgers and the new coupled MKdV equations with initial conditions. It can be concluded that this method is a very powerful and efficient technique in finding exact solutions for wide classes of problems. It is worth pointing out that the homotopy perturbation method presents rapid convergence solutions.

Acknowledgements

The authors wish to thank the referee for his suggestions and comments.

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